Location, Productivity, and Trade

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Major issue in trade: How does trade liberalization affect competition?

Competition has more than one dimension

- price competition
- similarity in product space

Location models allow us to examine both.
Motivation - Trade, Location, and Heterogeneity

Problem: Location models with significant heterogeneity become intractable

Trade models require differences in firm costs
- importers face extra costs
- reallocation gains from high-cost firm exit.

Contribution:
- New location model
  - allows arbitrary heterogeneity
  - tractable under a wide variety of preferences
- Analyze effect of a trade liberalization
Sequential Entry

I develop a location model with sequential entry

- Firms may enter after other firms.

- Critical feature: firms consider subsequent entry
  - high cost firms do not operate
  - low cost firms act to prevent further entry

- Simple Equilibrium
  - summarized by cutoff firm marginal cost $\bar{a}$
  - firm prices solve simple maximization problem
Trade Application

- Develop 2-country symmetric GE trade model
- Derive implications for competition after a trade liberalization
  - productivity gains through exit margin
  - reduction in markups
  - increase in distance between firms
- Discuss improvements on a Dixit-Stiglitz Model with CES preferences (Fixed Location Model):
  - small exporters
  - measured productivity gains from trade reform
Outline

Describe partial equilibrium location model
- present standard location environment
- introduce important difference in timing
- characterize the equilibrium

Provide simple trade application
- GE model with flexibly parameterized demand
- derive implications for trade reform
Players

The Game:

- Consumers
  - uniformly distributed around the circumference of a circle
  - demand goods which vary by location of producer

- Firms
  - located on the circle
  - sell to consumers around location
Strategies - Timing Overview

Timing is the critical feature of this model. Firms:

- Choose to enter in any stage $s \in \{1, 2, 3 \ldots, \infty\}$
- Firms entering in stage $s$ pick $(x_j, p_j)$ simultaneously
- Produce and earn profits after each stage
- Discount profits at rate $\delta$
An action for a firm is a triple: \((s_j, x_j, p_j)\)

A strategy is a mapping from histories up to stage \(s\) to actions.

I am looking for a standard subgame-perfect Nash equilibrium of this game.
Payoffs - Consumers

Consumer $i$ chooses

$$\max_{j,c_j} u(c_j, x_{ij}) - \lambda(p_j c_j)$$

- $c_j$ - quantity of good
- $x_{ij}$ - distance between consumer $i$ and firm $j$
- $p_j$ - price charged by firm $j$
- $\lambda$ - opportunity cost of expenditure

literature usually focuses on a special case
A Special Case

Consumer \( i \) inelastically demands one unit, chooses:

\[
\max_j \quad u - \eta \cdot x_{ij} - p_j
\]

- \( u \) - utility from consuming good, assumed large.
- \( \eta \) - linear distance cost

Consumer chooses lowest effective price

\[
p_j + \eta \cdot x_{ij}
\]
Classic Illustration

Effective Price

$p$

Demand Location

$\eta$

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Convex Costs

-Indirect Utility: $-V(p,x)$
Concave Costs

-Indirect Utility: $-V(p,x)$

![Diagram showing demand and location with concave costs.](image-url)
There are $N$ firms indexed by $j$.  

In each stage firms produce to satisfy demand earning

$$\pi_j = p_j y_j - a_j y_j - f$$

$$y_j = \ell_j / a_j$$

- $p_j$ - price
- $a_j$ - unit labor requirement
- $f$ - fixed cost of operating
Undercutting

A firm incurs a cost $\gamma$ at $s_j$ if it *undercuts* another firm.
Payoffs - Firms

Total profits for the firm are

\[ \pi_j = (1 - \delta) \sum_{s=s_j}^{\infty} \delta^s \left( (p_j - a_j) \cdot D - f \right) + \delta^{s_j} \gamma \]

- \( p_j \) - price
- \( a_j \) - unit labor requirement
- \( f \) - fixed cost of operating
- \( \gamma \) - penalty for undercutting another firm
Effective Price

Location

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An equilibrium

I consider a class of equilibria characterized by a cutoff marginal cost $\bar{a}$ and effective price $\bar{p}$.

- all firms with $a_j < \bar{a}$ enter in stage 1

- firms act to prevent the entry of firm $\bar{a}$
  - $\Rightarrow$ maximum effective price faced by any consumer $\bar{p}$

- deviations from equilibrium induce entry in stage 2
Equilibrium Prices

Effective Price

Location

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Firms in Equilibrium

Most firms pick prices to solve

$$\max_{p_j} \ (p_j - a_j) \cdot D(p_j, \bar{p})$$

The cutoff $\bar{a}$ is selected to be as low as possible consistent with firm rationality:

$$\sum_{j: a_j < \bar{a}} D(p_j^*, \bar{p}(\bar{a})) \geq 1$$
Effective Price

Location

\bar{p}, p
Effective Price

\[ \bar{p} \]

\[ p \]

\[ a \]

\[ \Pi \]

Effective Price

Location

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Effective Price

Location

Effective Price

\( \bar{p} \)

\( p \)

\( a \)

\( \Pi \)
Equilibrium Statement

The equilibrium is characterized by a cutoff firm $J$ and a marginal firm $K$ such that

- The boundary effective price between any two firms is $\bar{p} = a_J + 2\sqrt{\eta f}$

- Firm prices and demands, $(p_j, d_j)$ are given by
  
  \[
  \begin{cases}
  (\bar{p} + a_j)/2, & (\bar{p} - a_j)/\eta, \quad a_j < a_K; \\
  \text{intermediate } (p, d), & \text{firm } K; \\
  (2\bar{p} + a_j)/3\eta, & 2(\bar{p} - a_j)/3\eta, \quad a_K < a_j < a_J; \\
  \text{no entry}, & a_J \leq a_j.
  \end{cases}
  \]
Importance of Large Number of Firms

It is critical that there are many firms just above $\bar{a}$

Mechanism: if a firm tries to grab more demand, marginal firms subsequently enter.

To capture this idea, define

$$\varepsilon = \max_j a_{j+1} - a_j$$
Results

Existence

For any $A = \{a_1, \ldots, a_N\}, (f, \delta, \varepsilon)$, there exists an $\gamma$ such that this equilibrium exists.

Limiting Equilibrium

For any $\gamma > 0$ there exists there exists an $(f, 1 - \delta, \varepsilon)$ small enough to support this equilibrium.

As $\varepsilon \to 0$ we have $a_K \to a_J$. 
## Price Commitment - Comparison with Vogel 08

<table>
<thead>
<tr>
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<th>Price</th>
<th>Demand</th>
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<tbody>
<tr>
<td>$SE$</td>
<td>$(\bar{p} + a_j)/2$</td>
<td>$(\bar{p} - a_j)/\eta$</td>
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<tr>
<td>$V$</td>
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## Price Commitment - Comparison with Vogel 08

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<td>Specific Demand</td>
<td>Unrestricted Demand</td>
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<td>Fixed Prices</td>
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<td>$V$</td>
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Role of $\gamma$

Effective Price

$p$

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Role of $\gamma$

Effective Price

$p$

$\bar{p}$

Eq. Demand

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Role of $\gamma$

Effective Price

Location

$p$

Eq. Demand

Extra Demand

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Role of $\gamma$

Effective Price

$\bar{p}$

Eq. Demand

Subsequent Entrants

Location
Extension to Trade

- Develop 2-country symmetric GE trade model
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  - productivity gains through exit margin
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- Discuss improvements on a Dixit-Stiglitz Model with CES preferences:
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Limiting Equilibrium

For this application, consider the limiting equilibrium as

\[ f, 1 - \delta, \varepsilon, \gamma \to 0 \]

- appropriate for a large number of firms
- \( \bar{p} \to \bar{a} \)
- \( a_K \to \bar{a} \)
Consumers

There are a continuum of industries indexed by $\omega \in [0, 1]$.

Consumer $i$ chooses

$$\max_{j(\omega), c_i(\omega)} \int_0^1 \left( \frac{c_i(\omega)\rho - 1}{\rho} - \eta x_{ij}(\omega)^{\sigma} \right) d\omega$$

s.t. $$\int_0^1 p_{j(\omega)} c_i(\omega) d\omega \leq 1 + \Pi + T$$
Demand

There are two margins to demand

- intensive margin - individual demand $c(p, \bar{a})$
- extensive margin - consumers purchasing from the firm $d(p, \bar{a})$

Firms set prices as if they always capture consumers with

$$V(p, x) \geq V(\bar{a}, 0)$$
Firms

Domestic Firms choose

$$\max_p (p - a) \cdot c(p, \bar{a}) \cdot d(p, \bar{a})$$

Foreign Firms choose

$$\max_p (p(1 - \tau) - a) \cdot c(p, \bar{a}) \cdot d(p, \bar{a})$$

$$= \max_p (1 - \tau)(p - \frac{a}{1 - \tau}) \cdot c(p, \bar{a}) \cdot d(p, \bar{a})$$
Firm Distribution

Large number of firms $N$ in each country

- CDF approximated by $H(a)$, with density $h(a)$
- Pareto w/curvature $\kappa$

Symmetric two-country model with tariff $\tau$ identical to closed economy where

$$\tilde{N} = N(1 + (1 - \tau)^{\kappa})$$
Prices

Firms' prices are an implicit function of the first order condition

\[
\frac{p - a}{a/\rho - p} = \sigma \left(1 - \left(\frac{p}{a}\right)^{\frac{\rho}{1-\rho}}\right)
\]

- \( \rho > 0 \ \Rightarrow \ a < p < a/\rho \)
- \( \lim_{\sigma \to \infty} p = a/\rho \) - the Dixit-Stiglitz pricing rule
- \( \lim_{\sigma \to 0} p = a \) - the perfect competition pricing rule
Prices

Firms prices are an implicit function of the first order condition

\[
\frac{p - a}{a / \rho - p} = \sigma \left( 1 - \left( \frac{p}{\bar{a}} \right)^{\frac{\rho}{1-\rho}} \right)
\]

The firm’s pricing policies are

\[
p(a, \bar{a}) = a \cdot M \left( \frac{\bar{a}}{a} \right)
\]

Where \( M \) is a markup function with

- \( M(1) = 1 \)
- \( M' > 0 \)
Effects of a Trade Reform

Productivity

\[ \frac{\partial \bar{a}}{\partial \tau} > 0 \]

A reduction in tariffs reduces cutoff marginal costs, hence average costs.
Effects of a Trade Reform

Markups

\[ p(a, \bar{a}) = a \cdot M\left(\frac{\bar{a}}{a}\right) \]

So that markups decline as \( \bar{a} \) declines.

The simple average of markups, however, are constant

\[ \frac{1}{H(\bar{a})} \int_{0}^{\bar{a}} M(\bar{a}/a)da \]
Effects of a Trade Reform

Product Similarity

For $\rho > 0$ we have

$$\frac{\partial \tilde{N} \cdot H(\bar{a})}{\partial \tau} > 0$$

A reduction in tariffs reduces domestically available varieties
- new varieties are imported
- more domestic varieties exit

Average distance between firms increases!
Effects of a Trade Reform

Product Similarity and Welfare

Is competitive effect of a liberalization overstated?

Greater product separations is welfare improving

- location models feature excess variety
- greater distance means low cost firms serve more of the market
Small Exporters

Arkolakis (2008) discusses the failure of fixed-cost CES - Dixit-Stiglitz models to account for small exporters in French export data.

Fixed costs imply a minimum scale for operating firms.

Variable markups limit the set of operating firms without a minimum scale

\[
\lim_{p \to \bar{a}} d(p, \bar{a}) = 0
\]
Productivity Gains

With a CES / Dixit-Stiglitz specification prices are constant

$$p_t(a) = p_{t-1}(a) = a/\rho$$

GDP at current prices is fixed by the budget constraint

$$\forall t \quad N \int_A p_t(a)y_t(a)da = K$$

Real GDP is equal to GDP at current prices.

$$N \int_A p_{t-1}(a)y_{t-1}(a)da = N \int_A p_t(a)y_t(a)da = N \int_A p_{t-1}(a)y_t(a)da$$
Productivity Gains

The budget constraint also fixes GDP in current period prices in the location model

$$\forall t \quad N \int_{\mathcal{A}} p_t(a)y_t(a)da = K$$

But prices depend positively on the cutoff level $\bar{p}$ so that

$$\tau_{t-1} > \tau_t \quad \Rightarrow \quad \bar{a}_{t-1} > \bar{a}_t \quad \Rightarrow \quad aM(\bar{a}_{t-1}/a) > aM(\bar{a}_t/a)$$

So that

$$N \int_{\mathcal{A}} p_{t-1}(a)y_{t-1}(a)da = N \int_{\mathcal{A}} p_t(a)y_t(a)da < N \int_{\mathcal{A}} p_{t-1}(a)y_{t}(a)da$$
Conclusion

Sequential entry location model
- handle arbitrary differences in firm costs
- tractable under a flexibly parameterized utility function

2-country symmetric trade model
- declining average costs, markups, varieties
- improves on benchmark trade model in several dimensions
Extensions

Model
- fixed locations under trade reform
- variable consumer density - home bias
- fixed locations across countries

Trade Application
- quantitative version of the model
- asymmetric countries
- cost draws